

Asymmetry and the Cost of Capital

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Abstract

The expected cost of capital is a crucial component for most of the topics in corporate finance. Unfortunately in the presence of risky debt, it is systematically overestimated. This bias is increasing in leverage and the volatility of cash flows. We show the existence of the bias and assess its size. We finally propose a novel methodology to obtain a direct unbiased estimation of the expected return on assets. This method avoids the computational error that is obtained from the estimation of the individual components.

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The cost of capital is a crucial component in every corporate finance decision: it is mainly used to assess performance and to estimate value and provides a frame for the corporate decision-making process. In order to be sustainable, a firm would need to set the required return on operating assets –i.e. the cost of capital- equal to the return investors expect to derive from those assets. In theory, this figure should be linked to the probability distribution of the assets' returns, but this is not observable. What can be observed is, at most, the probability distribution of the returns generated by the securities that the company issued to finance the assets; namely, debt and equity. Therefore, it is not surprising that the standard procedure to measure the cost of capital relies on the measurement of the market returns of debt and equity.

Consider the situation of an all-equity firm; in this case the cost of capital is equal to the cost of equity, i.e. $K_A = K_e$, and since K_e is observable it is easy to calculate the cost of capital of the firm. The problem arises when a firm is financed with debt and equity; in such a case, the return on assets is not observable. In this situation, the most common way to approach the cost of assets, K_A is to compute it as the weighted average of the cost of equity K_e , and the post-taxes cost of debt $K_d \times (1 - t)$; where the weights result from the market valued share of equity and debt. The following equation summarizes the estimation of the cost of capital.

$$K_A = K_e \times \frac{E}{D + E} + K_d \times (1 - t) \times \frac{D}{D + E}$$

However, the cost of equity, K_e , and the cost of debt, K_d , -at least as they are usually estimated- are quite different in nature. On one side, analysts and practitioners tend to use the *promised* return on debt instead of the *expected* return; this is not a minor difference since this estimation would only coincide in a risk-free scenario; in other words, the *expected return* on debt would equal the *promised return* on debt only if the probability of default is equal to zero. It is therefore easy to see that the size of the bias introduced by this practice increases with leverage and the probability of default.

On the other side, the cost of equity, K_e , usually estimated using the Capital Asset Pricing Model (CAPM) assumes that returns follow a normal distribution.¹ As we will show in this paper, the presence of risky debt prevents normality of equity returns and therefore introduces a bias in the estimation of the cost of equity through the CAPM.

Therefore, two issues need further examination. First, the traditional computation of the weighted average cost of capital, which nowadays combines two ununiformed parameters –a promised cost of debt and the expected return on equity. Second, the expected return on equity, which is typically estimated assuming normality; assumption that is not valid in the presence of risky debt.

These issues have not been acknowledged in the finance literature for years. There are only few studies advancing some ideas on how to deal with the presence of risky debt. Kaplan and Stein (1990) mention the problem without providing a final answer, and more recently, Cooper and Davydenko (2007) discuss the same issue and propose a solution based on the Merton (1974) model. Unfortunately, we find their proposed solution to be misleading and confusing.

This paper contributes to the corporate finance literature by acknowledging and restating the nature of the problem, by measuring the bias resulting from the usual standard estimation procedure, and by proposing a novel solution towards the correct measurement of the firm's cost of capital.

More specifically, we show how the presence of risky debt leads traditional computations to a systematic bias in the estimation of the cost of capital, and provide intuition to observe how this bias grows with the level of risk (i.e. it increases with leverage and with the volatility of the firm's cash flows). After explaining the nature of the problem, we show the difference between the expected and promised cost of debt, the existence of the estimation bias, and a measure its relative importance using Montecarlo Simulation models. Finally, using an analytical and numerical methodology, we provide an alternative

¹ The formal assumption in CAPM is the assumption of the mean variance portfolio theory: which is that investors' utility function can be expressed in terms of mean and variance, i.e. higher moments are not taken into consideration by the investors. Regardless of the investor's utility function, if returns are normally distributed, the mean and variance contain all relevant information about the distribution. Thus assuming a normal distribution of assets is enough to meet the CAPM assumption.

–and correct- specification for obtaining the expected cost of capital –a direct estimation of the required return on operating assets.

The remainder of the paper is as follows: in Section 1 we discuss the asymmetries embedded in the promised cost of debt and its importance for the estimation of the cost of capital; in Section 2 we analyze the similarities and differences in the case of the cost of equity. In Section 3 we use Montecarlo Simulations to show the existence of the bias in the traditional estimation of the expected cost of capital, and in Section 4 we propose a novel methodology that estimates the cost of capital without the bias introduced by debt. Finally, Section 5 concludes.

1. The cost of Debt

Asymmetries in corporate returns go hand in hand with the use of debt in the capital structure; that is, in all-equity firms assets and equity returns can be assumed to be symmetric. Debt contracts, in contrast, have two possible outcomes: they can be either performed via payment of the promised cash flow, or defaulted; thus, they have a non-symmetric nature. It is important to notice that the lack of symmetry of debt returns is directly transferred to equity returns, since both should add up to symmetric assets returns. In other words, symmetric assets returns cannot be decomposed into a symmetric and a non-symmetric component. Therefore, a debt-equity firm has returns on assets that are assumed to be symmetric, which result from the combination of two non-symmetric components: debt and equity.

The asymmetry of debt returns causes some problems when estimating the firm's expected returns. These problems, however, are often overlooked. In fact, there are two common factual errors connected to this issue. First, in search of average returns, it is frequent to substitute the mean with the mode; mode and mean, however, only coincide in the case of symmetrical unimodal distributions. Second, taking for granted low probabilities of default, it is often assumed that the mode coincides with the promised payment to debt holders. Accordingly, mean returns are substituted by modal returns, which in turn are exchanged by promised payments. As a result, the promised (as opposed

to the expected) cost of debt is usually used as a proxy for the return on debt, typically referred as K_d .

Given its non-symmetric nature, a true expected return on debt should take into account not only the promised yield but also the expected default loss (probability of default *times* the recovery value of debt in default). The difference between expected and promised returns might be significant, since these two concepts only coincide if probability of default is equal to zero; otherwise, using the promised yield would overestimate the true cost of debt and, therefore, the true cost of capital. The size of this bias increases with the level of risk associated to debt, and with the cost of default.

This problem has been already acknowledged by Cooper and Davydenko (2007) – henceforth CD07–, who propose a solution for the estimation of the expected return of risky debt based on the Merton (1974) model. Even though we coincide in the general understanding of the problem, we find their solution somewhat incomplete and confusing.

Our first objection stems from the fact that, in our view, their approach suffers from a circularity problem. The authors get on the estimation of the expected cost of risky debt, in order to be able to compute the expected return on assets, which they state to be unobservable. In this process, they suggest computing the cost of risky debt as a promised yield minus some portion of the spread which is directly associated to the expected default, δ . However, to achieve the estimation of δ , they use the risk premium on assets, π , as one of the basic inputs. Yet, by definition, adding π to the risk-free rate is already sufficient to obtain the unobservable return on assets (defined to be equal to $r_f + \pi$); therefore, the estimation of the cost of risky debt, at least within this process, seems to be unnecessary.

This first concern may appear of little relevance. In fact, the authors have delivered a direct method to estimate K_A , by providing a formula that allows the computation of the risk premium on assets, π . Once π is obtained, one would simply need to add it to the corresponding risk-free rate. However, here comes the second difficulty: the estimation of the unobservable π .

The CD07 approach asserts that given that equity could be regarded as a call option on the firm's assets (which directly connects with the Merton Model), both equity and assets have the same underlying source of risk. Therefore, it assumes that the risk premium on assets, π , and equity, π_E , are linked by the following equation (summary of equation #10 in the CD07 paper):

$$\frac{\pi}{\pi_E} = \frac{\text{Volatility of Assets}}{\text{Volatility of Equity}}$$

However, it is important to notice that this equation assumes the risk premium to be proportional to volatility (σ), which is not the regular assumption in standard single-factor capital asset pricing models.² For example, the usual estimation of K_e using the CAPM, assumes that $\pi_e = \beta_e \times MRP$. Applying the same estimation to the assets, we end up with:

$$\frac{\pi}{\pi_e} = \frac{\beta \times MRP}{\beta_e \times MRP} = \frac{\beta}{\beta_e}$$

By definition

$$\beta_x = \rho_{x,M} \frac{\sigma_x}{\sigma_M}$$

Then

$$\frac{\pi}{\pi_e} = \frac{\beta}{\beta_e} = \frac{\rho_M \frac{\sigma}{\sigma_M}}{\rho_{e,M} \frac{\sigma_e}{\sigma_M}} = \frac{\rho_M \times \sigma}{\rho_{e,M} \times \sigma_e} \neq \frac{\sigma}{\sigma_e}$$

where MRP stands for market risk premium, ρ is the correlation coefficient and β is the usual beta estimate.

Thus, under the CAPM, the CD07's equation is only valid if the correlation between the returns of the asset and the market is equal to the correlation between equity and the market, which is clearly not true, since assets and equity behave completely different in case of default. The problem arises again from the asymmetry of debt contracts. Debt

² And the solution will be even more questionable if compared to multi-factor asset pricing models.

returns are non-symmetric; as stated before, if we assume that asset returns are symmetric, equity returns cannot be so.³

There is an additional –and potentially more serious– concern with respect to Equation 10 (in CD07). Their approach is based on the Merton model, which emphasizes the idea that equity contracts should be understood as an option on assets. Therefore, the suggestion of estimating the cost of equity by adding an equity risk premium (π_e) to a risk-free return (r_f) –which will then be used as a discounting factor– seems contradictory. In other words, it sounds odd to talk about the risk premium of an option. Therefore, the suggested estimation of π does not seem to provide the right answer, either.

So far, we have observed that common practice estimates the Weighted Average Cost of Capital (WACC) combining an expected return on equity, K_e , and a promised return on debt, K_d ; inconsistency that produces a figure that is neither an expected cost of capital nor a promised return. Before suggesting a potential solution to this problem, we go one step forward and show how the presence of risky debt also causes the estimation of the other parameter of the WACC –the expected return on equity– to be mistaken.

We take advantage of this following section in order to introduce a simulation framework that will contribute to gain intuition. Nevertheless, the proposed methodology would not rely in the estimation of individual components of the WACC, rather, it will suggest computing a direct estimate of the discount rate on the firm's operating assets.

³ The inconsistency of equation (10) could also be shown using the WACC formula. Simplifying the risk-free rate and using CD07's notation, the WACC formula becomes

$$\pi = \delta \times p_D + \pi_e \times (1 - p_D)$$

where p_D is the leverage ratio.

And using CD07's formula (11) $\pi = \frac{\pi_e \times (1 - p_D)}{N(d_1)}$, we obtain $\pi = \delta \times p_D + \pi \times N(d_1)$.

Then,

$$\delta = \pi \frac{(1 - N(d_1))}{p_D}$$

Which is evidently different from CD07's equation (12), according to which

$$\delta = -\left(\frac{1}{T}\right) \times \ln \left[e^{(\pi-s)T} \times \frac{N\left(-d_1 - \pi_e \times \frac{\sqrt{T}}{\sigma_e}\right)}{p_D} + N\left(d_2 + \frac{\pi_e \sqrt{T}}{\sigma_e}\right) \right].$$

2. The cost of Equity

If we consider that equity rights can be viewed as a call option on the firm's assets, with a strike equal to the amount of debt, we can notice that the distribution of equity returns can hardly be considered symmetric.⁴ Therefore, the fact that equity resembles option patterns makes unreasonable to use the CAPM to estimate the equity expected return.⁵ In other words, even if applying the CAPM to the return on equity when there is no debt seemed a plausible procedure; when leverage is significant, the behavior of the return on equity is quite different from a normal distribution, and applying the CAPM would lead to inconsistencies.

2.1. The True Expected Return on Equity and the CAPM

As we mentioned, equity returns of firms financed with debt and equity (as opposed to all-equity firms) are non-symmetric. At least, we could say that this is the case whenever there is limited responsibility (as we have in the case of incorporated firms).⁶ One should therefore estimate K_e based on the asymmetric distribution of returns. The problem, however, is that empirical estimations are not directly obtained from these asymmetric distributions, but more generally based on the estimated beta and build-into the equity returns using the CAPM ($K_e = r_f + \beta \times MRP$). The implicit assumption is that markets are in equilibrium and that the CAPM is a valid framework.

Can we assume this computation to be an adequate estimation of truly expected returns? We present a simulation approach to show that this is not the case. Moreover, to assess the difference we run a simulation and obtain the empirical estimation of the true expected return on equity and compare it with the empirical estimation based on the CAPM. In both cases *empirical estimation* means that they are computations based on

⁴ A profound solution to this issue requires a considerable change in standard practices –moving towards contingent claim analysis–, which are out of the scope of this paper.

⁵ Remember that CAPM is used to estimate the expected return assuming the market is in equilibrium.

⁶ If a company has unlimited responsibility, returns would not resemble option patterns; on the contrary, they would be indeed symmetric, assuming unlimited personal wealth.

simulated data. The basic process is as follows. Assuming return on assets are normally distributed, with a given asset beta and correlation coefficient –relative to market returns-, we obtain the expected return on assets, K_A . Assuming an expected return on debt, K_d^E , we obtain the necessary promised yields for different levels of risk –leverage-, K_d . The return on equity is then computed from the residual (given that the value of equity is the value of assets minus the value of debt), and through the estimation of the empirical levered beta and the use of the CAPM. We finally compare these results.

2.1.1. The Simulation Setting:

- Simulation of the capital market. We will assume the market index has a normal distribution, with mean of 11% and standard deviation of 20%.⁷

- Simulation of the asset returns (r_A). Similar to the market, we assume assets returns to be normally distributed. We assume assets to have a β equal to 1 and correlation, ρ , equal to 0.5 with respect to the market returns. Under these assumptions we compute expected assets returns, $K_A = 11\%$, and standard deviation or volatility, $\sigma = 40\%$.

- Return of debt, r_d . The return of debt will be equal to the promised payments, K_d , if there is no default, or to a lower figure, in case of default. More specifically,

$$r_d = \begin{cases} K_d & \text{if } (1 + r_A) \geq (1 + K_d) \times p_D \\ \frac{(1 + r_A)}{p_D} - 1 & \text{if } (1 + r_A) < (1 + K_d) \times p_D \end{cases}$$

where p_D is the leverage ratio, and the rest of variables as previously defined.

The simulation requires assigning different K_d , according to different leverage ratios. We chose K_d so that the expected return to debt K_d^E is maintained. That is, we consider the financial creditor to adjust K_d at increasing risk levels so that the expected return remains constant. In our simulation we assume K_d^E to be equal to 5.8%.⁸

⁷ That is equivalent to assume a risk free rate, $r_f=5\%$, and the Equity Market Risk Premium, $MRP=6\%$.

⁸ Obtained adding the r_f rate (5%) to the corresponding spread; where the spread is proxy by the average spread of corporate bonds of non-financial institutions of the last 150 years, estimated by Giesecke, K., Longstaff, F. A., Schaefer, S. M., and Strebulaev, I. A. 2010. See Corporate Bond Default Risk: a 150-year Perspective. SSRN eLibrary).

- Simulation of the return on equity, r_e . The value of equity will be the value of assets minus the value of debt. In case of default, such a value accounts to zero. That is, in default scenarios r_e would be equal to -1. Formally,

$$r_e = \begin{cases} \frac{r_A - K_d \times p_D}{1 - p_D} & \text{if } (1 + r_A) \geq (1 + K_d) \times p_D \\ -1 & \text{if } (1 + r_A) < (1 + K_d) \times p_D \end{cases}$$

2.1.2. The Simulation

We analyze the impact of debt on the proper estimation of equity returns: indeed, we are interested in computing the distortion that results when default scenarios are not negligible. We consider two extreme scenarios; one with minimum leverage and other with high leverage ratios.

As a first step, we compute the simulated assets and equity expected returns (obtained from their corresponding simulated distributions; i.e. taking the arithmetic average of returns). Then, we estimate K_e using the CAPM, taking as an input the beta estimation that we obtain from the generated sample (here referred as *empirical* beta). We refer to this cost of equity estimation as K_e^β . Finally, we compare these results.

The first scenario considers a firm with a leverage ratio of only 1%. In this case, the probability of default is 0.30%⁹. The K_d to make $K_e^e = 5.80\%$ is 6.10%.

⁹ In fact, the cumulative probability of having return on assets lower than -100% is of about 0,28 in our setting; therefore, this is fairly close. Actually, the fact that returns on assets cannot really be of less than -100% means that we should truncate our distribution at that level of returns; this would consequently affect the expected average return. Nevertheless, given that under assumptions this is a small probability, we would not take this asymmetry into account in our analysis.

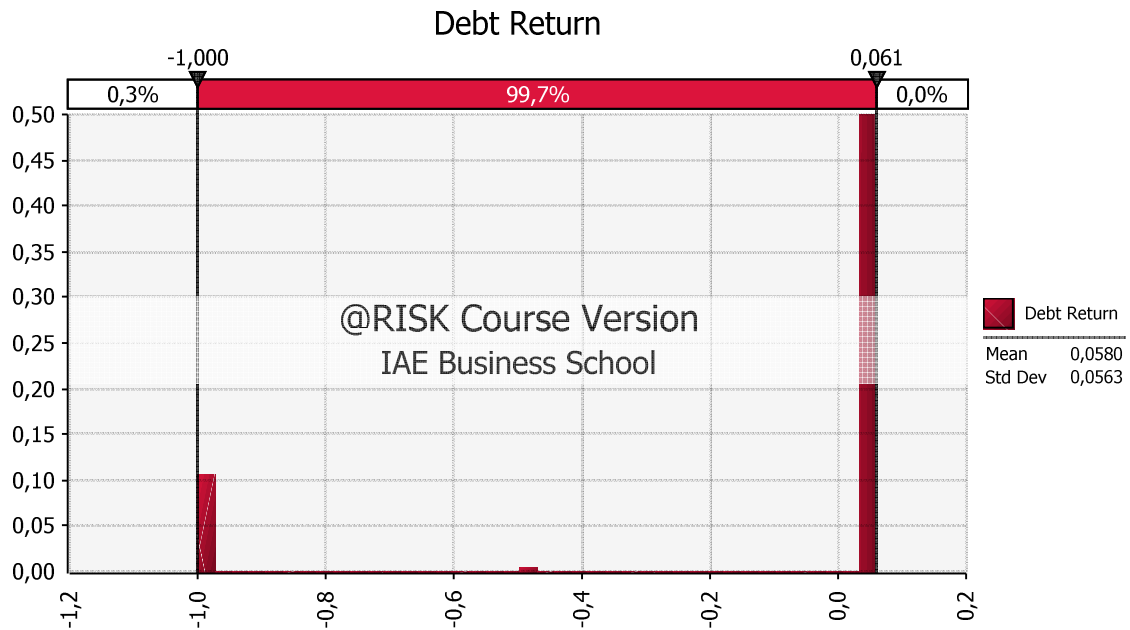


Figure 1. Distribution of debt return. Assuming 1% of leverage. Y axis has been truncated.

We are initially interested in the comparison between the simulated expected return on equity and the estimation that would result based on the CAPM model. The former, gives an 11.09% return. To compute the CAPM based estimation, we take the generated sample and estimate the *empirical* β ¹⁰–, which in turn corresponds to $K_e^\beta = 11.04\%$. The difference between these two estimations is not substantial at this stage. The reason is that when leverage is non-significant, the resulting asymmetry and its effects, are negligible.

¹⁰ This parameter is unstable and depends on correlations assumptions.

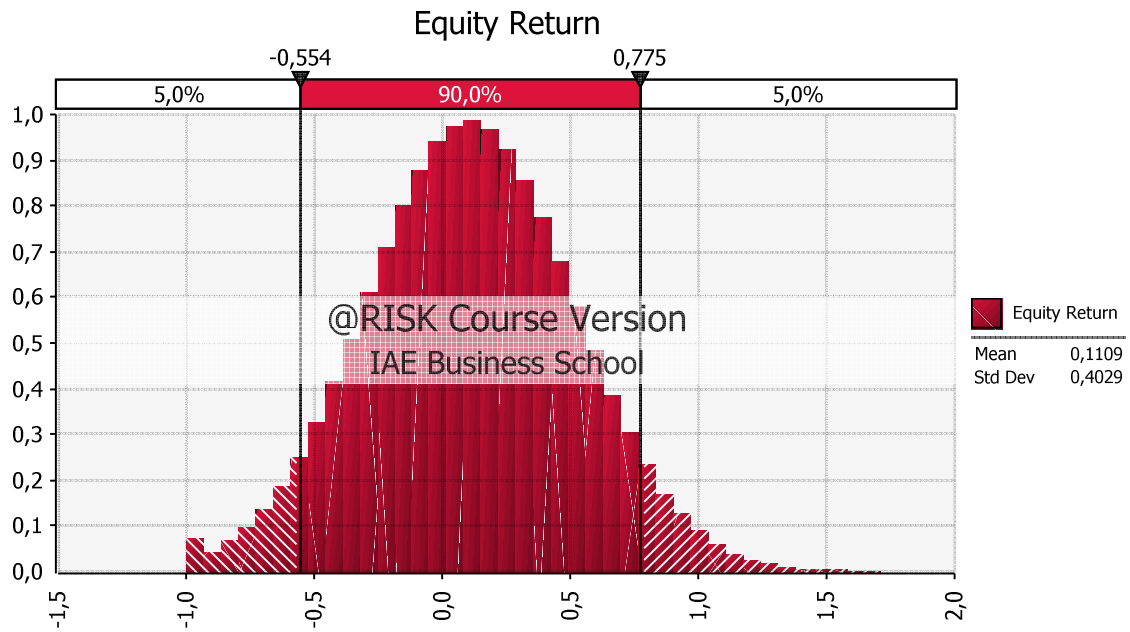


Figure 2. Distribution of equity returns. Assuming 1% leverage.

Under the second scenario, we assume a leverage ratio of 60%. In this case, the probability of default is 13.07% and the required K_d (to reach an expected return of 5.80%) is equal to 10.11%. The simulation output is presented in Figures 3 and 4.

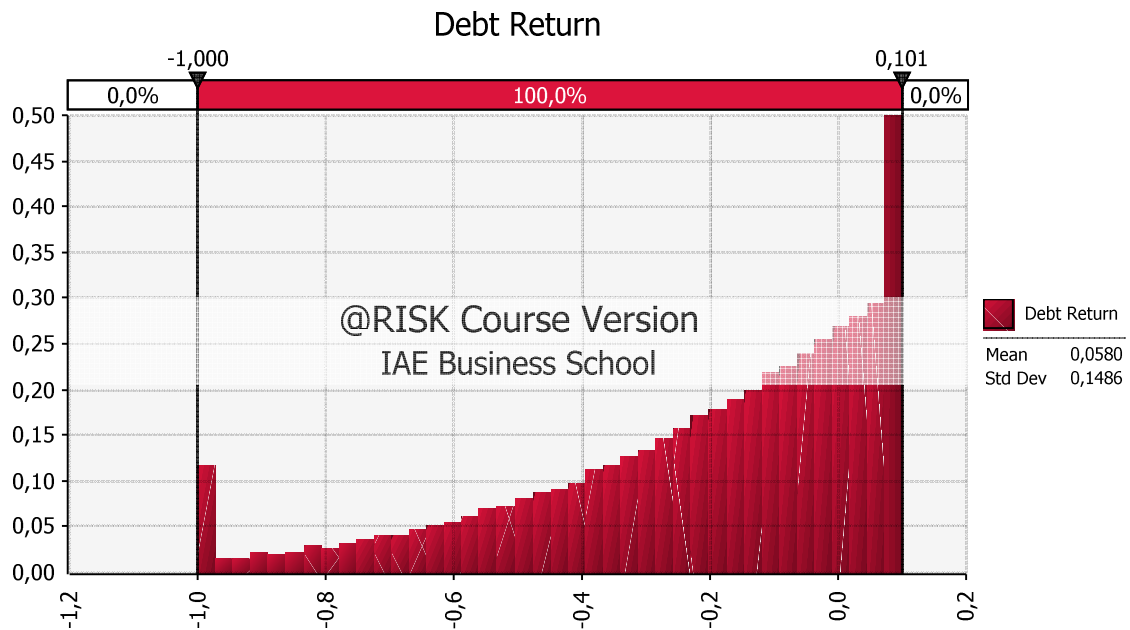


Figure 3. Distribution of debt return. Assuming 60% of leverage. Y axis has been truncated.

Now, let's put everything together and get a sense of how large could be the size of the resulting bias.

We will consider two extreme scenarios of low and high leverage (1% and 60%, respectively) and observe what happens with the estimated WACC using the standard practice, and how this result differs from the theoretical figure. To follow a more comprehensive analysis, we show not only the distribution of the returns on assets (blue line in the graph), but also the equity returns (red bars in the graph).

In the 1% leverage case, we observe that the distributions of assets and equity returns look very similar; expected returns are 11.03% and 11.09%, respectively. There is also only a tiny difference between the corresponding resulting volatilities (40.29% and 39.90%, respectively). All this is very natural, since this is *almost* an all-equity firm.

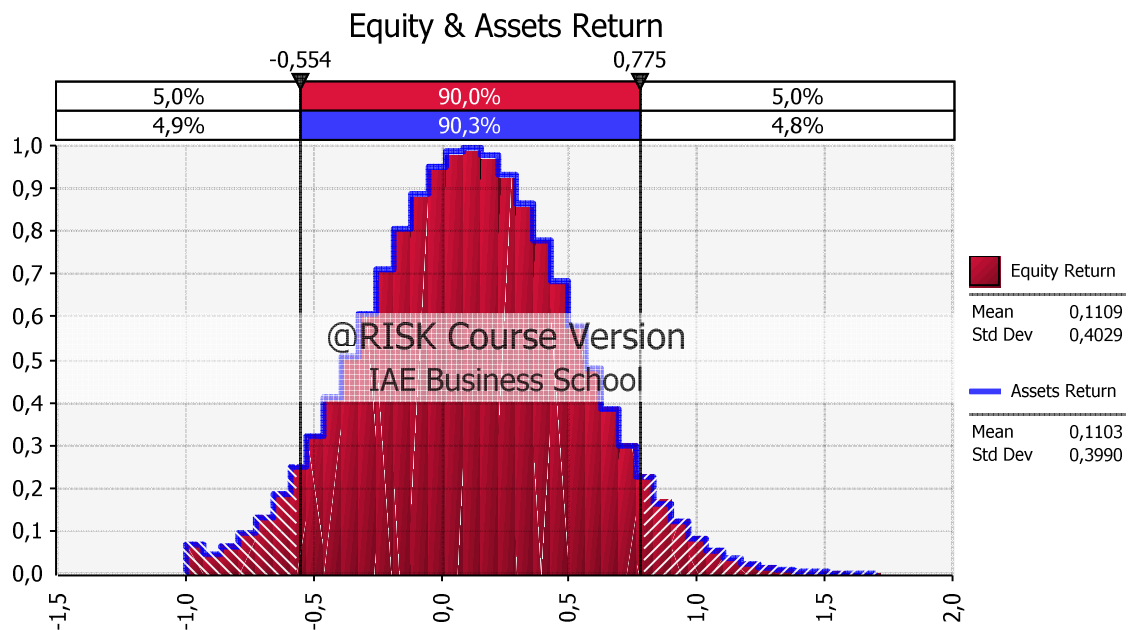


Figure 5. Distribution of assets and equity returns. Assuming 1% leverage.

Moreover, with the generated sample we obtained a beta estimate of 1.0069 and a corresponding K_e^β of 11.04%. If we combine this figure with the (promised) return of debt, K_d , 6.10%, we get to a WACC of 10.99%. Comparing this number with the theoretical return on assets (11%, which results from a risk-free rate of 5% plus a 6% of market risk

premium and $\beta = 1$), we observe the difference is fairly small. This is the case, given that the leverage ratio is immaterial and the asymmetry it causes is therefore negligible.

If we move to the scenario of high leverage ratio, the asymmetry becomes more relevant, with a probability of default of 13.07%. As a consequence, the volatility of equity returns are now significantly larger than the volatility of asset returns (88.96% versus 39.9%, respectively), and the mean return is more towards the right side, having reached 11.03% for assets and 18.88% for equity returns.¹¹

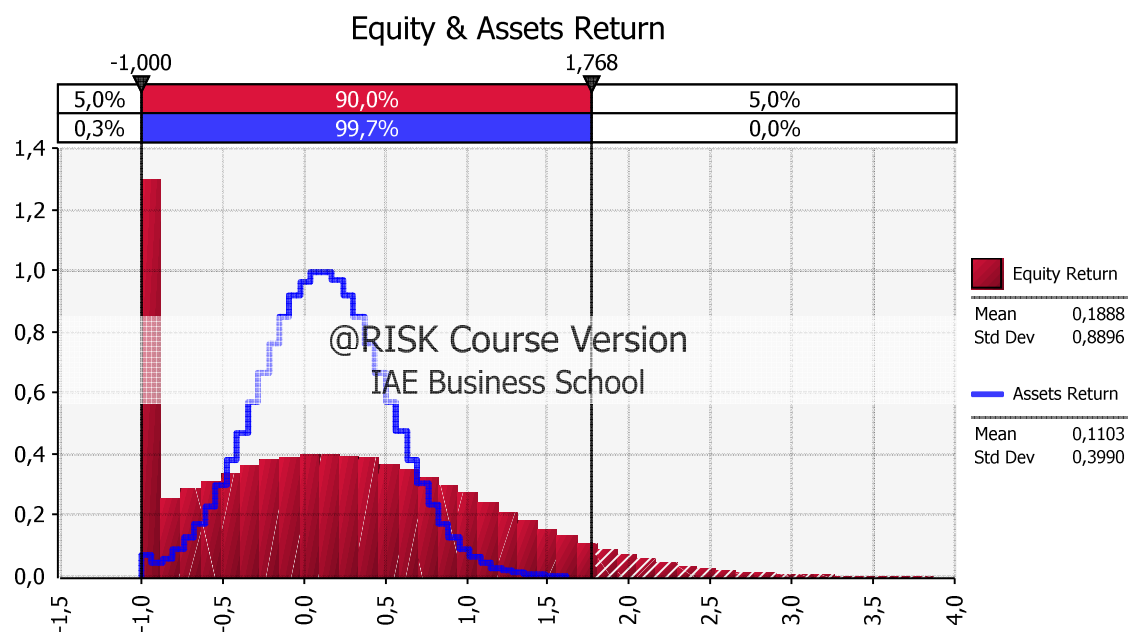


Figure 6. Distribution of assets and equity returns. Assuming 60% leverage.

To assess the distortive effect of asymmetries in the estimation of the expected return on assets, we can again compare the theoretical WACC versus the simulated *empirical* version. Theoretical WACC is equal to 11%, based on our initial assumptions. Now, from our empirical estimation we obtain a β estimate that used to estimate the K_e , and combined with a K_d of 10.11% produces a WACC of 13.27%; about 20% higher than the theoretical value.

¹¹ The small difference between 11 and 11.03% return on assets results from an asymmetry on assets returns that we decided not to explore here –being fairly small and outside the main scope in the discussion.

4. The Proposed Methodology

Our solution arises from looking at the problem from a different perspective. Instead of trying to estimate K_A as a weighted average of expected returns of debt and equity, we propose to identify the underlying normal distribution of asset returns, which is behind the observable distributions of debt and equity.

We know that the expected returns of equity and debt are a function of the expected return on assets –in other words, r_e and r_d are functions of the random variable r_A –, which in turn is known to have a normal distribution, with parameters K_A and σ_A . Thus, any statistical function of r_e and r_d are functions of K_A and σ_A . The methodology we propose consists of solving for K_A and σ_A in a system of equations that describes observable statistical functions of r_e and r_d .

Since the standard deviation of r_e is usually known –it is used to estimate β –, we may establish the first equation of the system based on its statistical definition of σ_e as¹²

$$\sigma_e = E(r_e^2) - E^2(\bar{r}_e) = \int_{-\infty}^{\infty} r_e^2 f(r_A) dr_A - \left(\int_{-\infty}^{\infty} r_e f(r_A) dr_A \right)^2$$

where $f(r_A) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_A - K_A)^2}{2\sigma_A^2}}$, the density function of the Normal distribution.

Given values to the parameters as in the second scenario we described ($\sigma_e = 0.8897$, $K_d = 10.11\%$ and $p_D = 60\%$), we can obtain a solution for this part.

Nevertheless, in order to find the solution for our two unknowns $\{K_A, \sigma_A\}$ we need another equation. In this case, we assume the probability of default (δ) to be known. The probability of default can be expressed as

¹² As r_e is a function defined in segments we could restate the equation as

$$\sigma_e = \int_{-\infty}^{(1+K_d)p_D-1} f(r_A) dr_A + \int_{(1+K_d)p_D-1}^{\infty} \left(\frac{r_A - K_d p_D}{1 - p_D} \right)^2 f(r_A) dr_A - \left(- \int_{-\infty}^{(1+K_d)p_D-1} f(r_A) dr_A + \int_{(1+K_d)p_D-1}^{\infty} \left(\frac{r_A - K_d p_D}{1 - p_D} \right) f(r_A) dr_A \right)^2$$

$$\delta = \int_{-\infty}^{(1+K_d)p_D-1} f(r_A) dr_A$$

which gives us a second equation involving our unknown variables. For this equation, we need to apply the same figures as before and add the probability of default obtained for the same scenario –i.e. $\delta = 13.07\%$.

The system of equations can be solved numerically to obtain the values searched. In the example of the second scenario (60% leverage ratio) we obtained the values: $K_A = 10.99\%$ and $\sigma_A = 40.00\%$, which are almost identical to the original values we used to make example.

According to this solution we do not need to estimate the discount rate of operating assets based on a combination of two observable inputs –returns on debt and equity-, since the asymmetry of debt returns leads to inconsistencies in such a procedure. Rather, this solution suggests using observable parameters of debt and equity returns to identify the underlying distribution of the return on operating assets. That is, we obtain a system of two equations that allows us to directly estimate the expected return on operating assets. The method requires to plug in the corresponding inputs into the system of equations –as provided above: the volatility of equity return, σ_e , the promised cost of debt, K_d , the leverage ratio, p_D , and the probability of default, δ – and solve for the return on operating assets, K_A , and the volatility of those returns, σ_A .

5. Conclusions

In this paper we discussed the implications of the asymmetries in the returns of risky debt for the estimation of the cost of capital. We highlight the difference between *promised* and *expected* returns to debt holders, and explain how the asymmetry of debt returns compromises the standard practice for the estimation of the cost of capital of the firm, in which the promised return to debt holders is used instead of the corresponding expected return. We continued by reporting how the asymmetry of debt causes a subsequent problem in the estimation of the expected cost of equity; more specifically, we

observe how the assumption of normal returns to equity holders is not fulfilled in the presence of risky debt. The conjunction of these two problems causes an upward bias in the estimation of the cost of capital performed using the standard procedures.

After an intuitive and analytical discussion of the issue, we use Montecarlo Simulation models to give a sense of its quantitative relevance. We show that in the presence of risky debt the usual methodologies overestimate the cost of capital of the firm, and that this bias is increasing in leverage and risk.

We conclude the paper by proposing a novel approach to solve this problem by taking a new perspective. We depart from the usual procedure of estimating the individual components of the weighted average cost of capital (i.e. cost of debt and equity), and attempt a direct estimation of the return on assets. The procedure suggests identifying the underlying normal distribution of the returns of the operational asset of the firm which is behind the observable distributions of debt and equity. We developed an example that illustrates this proposal.

The paper analyzes the problems behind the standard approach to the estimation of the cost of capital and suggests a new perspective. More research is needed in order to operate this solution to alternative contexts.

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